

1 Lösungen

1. Fasse zusammen !

a) $\underline{\underline{4x + 4y = 4(x + y)}}$

c) $\underline{\underline{5b + 5 = 5(b + 1)}}$

e) $\underline{\underline{pq - qr = q(p - r)}}$

b) $\underline{\underline{6p - 6q = 6(p - q)}}$

d) $\underline{\underline{pq + qr = q(p + r)}}$

f) $\underline{\underline{ax - bx - cx = x(a - b - c)}}$

2. Fasse zusammen !

a) $\underline{\underline{15m - 18n = 3(5m - 6n)}}$

c) $\underline{\underline{4u - 6v = 2(2u - 3v)}}$

e) $\underline{\underline{x^2 + 2x = x(x + 2)}}$

g) $\underline{\underline{5ax + 5bx = 5x(a + b)}}$

i) $\underline{\underline{49r^3 - 39r = r(49r^2 - 39)}}$

k) $\underline{\underline{54u + 81v + 108w = 9(6u + 9v + 12w)}}$

m) $\underline{\underline{8cx + 24dx - 12ex = 4x(2c + 6d - 3e)}}$

b) $\underline{\underline{4r + 12s = 4(r + 3s)}}$

d) $\underline{\underline{p^2 - pq = p(p - q)}}$

f) $\underline{\underline{a^2 - a = a(a - 1)}}$

h) $\underline{\underline{7by - 14bz = 7b(y - 2z)}}$

j) $\underline{\underline{5x - 10y + 15z = 5(x - 2y + 3z)}}$

l) $\underline{\underline{uv - u^2 + uw = u(v - u + w)}}$

n) $\underline{\underline{p^2 - pq + pr - p = p(p - q + r - 1)}}$

3. Fasse zusammen !

a) $\underline{\underline{24p^2 + 16pq = 8p(3p + 2q)}}$

c) $\underline{\underline{65e + 78e^2 = 13e(5 + 6e)}}$

e) $\underline{\underline{34y^4 + 17y^2 = 17y^2(y^2 + 1)}}$

g) $\underline{\underline{63x^3 + 21x^2 + 70x = 7x(9x^2 + 3x + 10)}}$

h) $\underline{\underline{52p^3q + 94p^2q^2 - 36pq^3 = 2pq(26p^2 + 47pq - 18q^2)}}$

b) $\underline{\underline{20u^2 - 55u = 5u(4u - 11)}}$

d) $\underline{\underline{42z^3 + 78z^2 = 6z^2(7z + 13)}}$

f) $\underline{\underline{9ax + 57ax^2 = 3ax(3 + 19x)}}$

4. Klammere (-1) aus !

a) $\underline{\underline{-x - y = -(x + y)}}$

c) $\underline{\underline{4a - 3b = -(-4a + 3b)}}$

e) $\underline{\underline{-c + 1 = -(c - 1)}}$

g) $\underline{\underline{-8a + 4b - 7c = -(8a - 4b + 7c)}}$

i) $\underline{\underline{4.5y + z - 0.8 = -(-4.5y - z + 0.8)}}$

b) $\underline{\underline{-2p + q = -(2p - q)}}$

d) $\underline{\underline{6u + 5v = -(-6u - 5v)}}$

f) $\underline{\underline{3y^2 - 5z^2 = -(3y^2 + 5z^2)}}$

h) $\underline{\underline{-2x^2 - 15x + 9 = -(2x^2 + 15x - 9)}}$

5. Fasse soweit wie möglich zusammen !

a) $\underline{\underline{(x + y)p + (x + y)q = (x + y)(p + q)}}$

c) $\underline{\underline{a(m - n) + 2(m - n) = (m - n)(a + 2)}}$

e) $\underline{\underline{p(x + y) + x + y = (x + y)(p + 1)}}$

f) $\underline{\underline{2a(5v + 10w) + 4b(10w + 5v) = (10w + 5v)(2a + 4b) = 5(2w + v)2(a + 2b) = 10(2w + v)(a + 2b)}}$

g) $\underline{\underline{52a^3b^2(3x + 6y) - 78ab^4(3x + 6y) = (3x + 6y)(52a^3b^2 - 78ab^4) = 3(x + 2y)13ab^2(4a^2 - 6b^2) = 39ab^2(x + 2y)(4a^2 - 6b^2) = 78ab^2(x + 2y)(2a^2 - 3b^2)}}$

h) $\underline{\underline{66pq(m - 1) + 102qr(m - 1) - 48pr(m - 1) = (m - 1)(66pq + 102qr - 48pr) = 6(m - 1)(11pq + 17qr - 8pr)}}$

6. Fasse soweit wie möglich zusammen !

- a) $\underline{\underline{(2a-b)(x+y) + 3a(x+y) = (x+y)(2a-b+3a) = (x+y)(5a-b)}}$
 b) $\underline{\underline{(5m+6n)(a-b) - 4n(a-b) = (a-b)(5m+6n-4n) = (a-b)(5m+2n)}}$
 c) $\underline{\underline{(8a-5b)(u+v) + (3a+4b)(u+v) = (u+v)(8a-5b+3a+4b) = (u+v)(11a-b)}}$
 d) $\underline{\underline{(7p+9q)(x-y) + (5p-6q)(x-y) = (x-y)(7p+9q+5p-6q) = (x-y)(12p+3q)}}$
 e) $\underline{\underline{(8a-5b)(u+v) + 3a(u+v) + 4b(u+v) = (u+v)(8a-5b+3a+4b) = (u+v)(11a-b)}}$
 f) $\underline{\underline{(8c+12d)(u-2v) - (7c-2d)(u-2v) = (u-2v)(8c+12d-7c+2d) = (u-2v)(c+14d)}}$

7. Fasse soweit wie möglich zusammen !

- a) $\underline{\underline{(2x+2y)(2x-2y) = 4(x+y)(x-y)}}$ b) $\underline{\underline{(5a+5b)^2 = (5(a+b))^2 = 25(a+b)^2}}$
 c) $\underline{\underline{(3p-6q)^2 = (3(p-2q))^2 = 9(p-2q)^2}}$
 d) $\underline{\underline{(18a-30b)(18a+30b) = 36(3a-5b)(3a+5b)}}$
 e) $\underline{\underline{(6x^2-8x+2)^2 = (2(3x^2-4x+1))^2 = 4(3x^2-4x+1)^2}}$
 f) $\underline{\underline{(2v-2w)^3 = (2(v-w))^3 = 8(v-w)^3}}$
 g) $\underline{\underline{(9x+3)^3 = (3(3x+1))^3 = 27(3x+1)^3}}$ h) $\underline{\underline{(4(a-b))^4 = 256(a-b)^4}}$
 i) $\underline{\underline{(ab+ac)^2 = (a(b+c))^2 = a^2(b+c)^2}}$ j) $\underline{\underline{(u(3u-12))^3 = u^3(3u-12)^3}}$

8. Fasse soweit wie möglich zusammen !

- a) $\underline{\underline{bm+bn+cm+cn = b(m+n) + c(m+n) = (b+c)(m+n)}}$
 b) $\underline{\underline{xu-yu+xv-yv = u(x-y) + v(x-y) = (u+v)(x-y)}}$
 c) $\underline{\underline{gv+gw-hv-hw = g(v+w) - h(v+w) = (g-h)(v+w)}}$
 d) $\underline{\underline{aq-bq-ar+br = q(a-b) - r(a-b) = (q-r)(a-b)}}$
 e) $\underline{\underline{14mp+14mq-9np-9nq = 14m(p+q) - 9n(p+q) = (14m-9n)(p+q)}}$
 f) $\underline{\underline{5cu-8cv+5du-8dv = c(5u-8v) + d(5u-8v) = (c+d)(5u-8v)}}$
 g) $\underline{\underline{amp-amq-anq+anp = am(p-q) - an(q-p) = am(p-q) + an(p-q) = (am+an)(p-q) = a(m+n)(p-q)}}$
 h) $\underline{\underline{ax+ay+bx+by+cx+cy = a(x+y) + b(x+y) + c(x+y) = (a+b+c)(x+y)}}$
 i) $\underline{\underline{mx+nx+px-m-n-p = x(m+n+p) - (m+n+p) = (x-1)(m+n+p)}}$

9. Fasse soweit wie möglich zusammen ! (doppelt)

10. Fasse soweit wie möglich zusammen !

- a) $\underline{\underline{21ax+35ay+20y+12x = 7a(3x+5y) + 4(5y+3x) = (3x+5y)(7a+4)}}$
 b) $\underline{\underline{32pv^2-36qv^2-80pw^2+90qw^2 = 4v^2(8p-9q) - 10w^2(8p-9q) = (8p-9q)(4v^2-10w^2) = 2(8p-9q)(2v^2-5w^2)}}$
 c) $\underline{\underline{175ax+75ay-35bx-15by = 25a(7x+3y) - 5b(7x+3y) = (7x+3y)(25a-5b) = 5(7x+3y)(5a-b)}}$
 d) $\underline{\underline{88ax^3-22ax^2+132bx^3-33bx^2 = 22ax^2(4x-1) + 33bx^2(4x-1) = (4x-1)(22ax^2+33bx^2) = 11x^2(4x-1)(2a+3b)}}$

$$e) \frac{10a^2bx^2 + 4ab^2x^2 - 40a^2by^2 - 16ab^2y^2}{8aby^2} = \frac{2abx^2(5a+2b) - 8aby^2(5a+2b)}{8aby^2} = \frac{(5a+2b)(2abx^2 - 8aby^2)}{8aby^2} = \frac{(5a+2b)(x^2 - 4y^2)}{2ab(5a+2b)} = \frac{2ab(5a+2b)(x+2y)(x-2y)}{2ab(5a+2b)(x+2y)(x-2y)}$$

11. Fasse soweit wie möglich zusammen !

$$a) \frac{g^2 + h^2 + 2gh}{(g+h)^2}$$

$$b) \frac{x^2 - 2x + 1}{(x-1)^2}$$

$$c) \frac{4n^2 + 4np + p^2}{(2n+p)^2}$$

$$d) \frac{p^2 - 8p + 16}{(p-4)^2}$$

$$e) \frac{m^2 + 26m + 169}{(m+13)^2}$$

$$f) \frac{\frac{4}{9}a^2 + ab + \frac{9}{16}b^2}{\left(\frac{2}{3}a + \frac{3}{4}b\right)^2}$$

$$g) \frac{a^2 + a + \frac{1}{4}}{\left(a + \frac{1}{2}\right)^2}$$

$$h) \frac{25p^4 + 80p^2q^2 + 64q^4}{(5p^2 + 8q^2)^2}$$

$$i) \frac{y^4 - 20y^2 + 100}{(y^2 - 10)^2}$$

$$j) 4m^2 + \frac{1}{4}n^2 - mn \rightarrow \underline{\text{bleibt}}$$

$$k) \frac{15x^2 + 30xy + 15y^2}{15(x^2 + 2xy + y^2)} = \frac{15(x+y)^2}{15(x+y)^2}$$

$$l) \frac{12r^2 + 75s^2 - 60rs}{3(4r^2 + 25s^2 - 20rs)} = \frac{3(2r-5s)^2}{3(2r-5s)^2}$$

12. Fasse soweit wie möglich zusammen !

$$a) \frac{m^2 - n^2}{(m+n)(m-n)}$$

$$b) \frac{100x^2 - 36}{(10x+6)(10x-6)}$$

$$c) \frac{1 - 4z^2}{(1-2z)(1+2z)}$$

$$d) \frac{\frac{1}{25}m^2 - \frac{9}{4}n^2}{\left(\frac{1}{5}m + \frac{3}{2}n\right)\left(\frac{1}{5}m - \frac{3}{2}n\right)}$$

$$e) \frac{9x^4 - y^4}{(3x^2 + y^2)(3x^2 - y^2)}$$

$$f) \frac{6g^2 - 6h^2}{6(g^2 - h^2)} = \frac{6(g+h)(g-h)}{6(g+h)(g-h)}$$

$$g) \frac{63v^2 - 28w^2}{7(9v^2 - 4w^2)} = \frac{7(3v+2w)(3v-2w)}{7(3v+2w)(3v-2w)}$$

$$h) \frac{u^8 - v^8}{(u^4 + v^4)(u^4 - v^4)}$$

$$i) \frac{a - a^3}{a(1 - a^2)} = \frac{a(1 + a)(1 - a)}{a(1 + a)(1 - a)}$$

$$j) \frac{x^3 - 100x}{x(x^2 - 100)} = \frac{x(x+10)(x-10)}{x(x+10)(x-10)}$$

$$k) \frac{4x^5 - 100x^3}{4x^3(x^2 - 100)} = \frac{4x^3(x+10)(x-10)}{4x^3(x+10)(x-10)}$$

13. Fasse soweit wie möglich zusammen !

$$a) \frac{(x+y)^2 - z^2}{(x+y+z)(x+y-z)}$$

$$b) \frac{p^2 + 2pq + q^2 - r^2}{(p+q)^2 - r^2} = \frac{(p+q+r)(p+q-r)}{(p+q+r)(p+q-r)}$$

$$c) \frac{x^2 - (y+z)^2}{(x+y+z)(x-y-z)}$$

$$d) \frac{u^2 - v^2 - 2vw - w^2}{u^2 - (v+w)^2} = \frac{(u+v+w)(u-v-w)}{(u+v+w)(u-v-w)}$$

$$e) \frac{a^2 + b^2 - c^2 + 2ab}{(a+b)^2 - c^2} = \frac{(a+b+c)(a+b-c)}{(a+b+c)(a+b-c)}$$

$$f) \frac{49b^2 + 25c^2 - 70bc - 36d^2}{(7b-5c)^2 - 36d^2} = \frac{(7b-5c+6d)(7b-5c-6d)}{(7b-5c+6d)(7b-5c-6d)}$$

$$g) \frac{r^2 - s^2 - t^2 + 2st}{r^2 - (s-t)^2} = \frac{(r+s-t)(r-s+t)}{(r+s-t)(r-s+t)}$$

14. Fasse soweit wie möglich zusammen !

$$a) \frac{x^2 + 7x + 12}{(x+4)(x+3)}$$

$$b) \frac{a^2 + 13a + 40}{(a+5)(a+8)}$$

$$\begin{array}{ll} \text{c) } \underline{\underline{z^2 + 20z + 96 = (z + 8)(z + 12)}} & \text{d) } \underline{\underline{m^2 + 5m + 6 = (m + 2)(m + 3)}} \\ \text{e) } \underline{\underline{p^2 + 5p + 4 = (p + 4)(p + 1)}} & \text{f) } \underline{\underline{b^2 + 10b + 9 = (b + 9)(b + 1)}} \\ \text{g) } \underline{\underline{x^2 + x - 12 = (x - 3)(x + 4)}} & \text{h) } \underline{\underline{x^2 - x - 12 = (x + 3)(x - 4)}} \\ \text{i) } \underline{\underline{x^2 - 7x + 12 = (x - 3)(x - 4)}} & \text{j) } \underline{\underline{x^2 + 8x + 15 = (x + 5)(x + 3)}} \end{array}$$

15. Fasse soweit wie möglich zusammen !

$$\begin{array}{l} \text{a) } \underline{\underline{9a^2 - (a - b)^2 = (3a + a - b)(3a - a + b) = (4a - b)(2a + b)}} \\ \text{b) } \underline{\underline{(a - b)x^4 + (b - a)x^2 = (a - b)x^4 - (a - b)x^2 = (a - b)x^2(x^2 - 1) = (a - b)x^2(x + 1)(x - 1)}} \\ \text{c) } \underline{\underline{9p^4(a - b) - 25q^2(a - b) = (a - b)(9p^4 - 25q^2) = (a - b)(3p^2 + 5q)(3p^2 - 5q)}} \\ \text{d) } \underline{\underline{m^2 - n^2 - p^2 + 2np = m^2 - (n^2 + p^2 - 2np) = m^2 - (n - p)^2 = (m + n - p)(m - n + p)}} \\ \text{e) } \underline{\underline{100x^2 - 4(7x - 2y)^2 = (10x + 2(7x - 2y))(10x - 2(7x - 2y)) = (10x + 14x - 4y)(10x - 14x + 4y) = (24x - 4y)(-4x + 4y) = 4(6x - y)4(-x + y) = 16(6x - y)(y - x)}} \\ \text{f) } \underline{\underline{48(a + b)^2 - 12(a - b)^2 = 12(4(a + b)^2 - (a - b)^2) = 12(2(a + b) - (a - b))(2(a + b) + (a - b)) = 12(2a + 2b - a + b)(2a + 2b + a - b) = 12(a + 3b)(3a + b)}} \end{array}$$

16. Kürze soweit wie möglich !

$$\begin{array}{ll} \text{a) } \frac{a + b + c}{a + b + d} \rightarrow \text{bleibt} & \text{b) } \frac{a \cdot b \cdot c}{a \cdot b \cdot d} = \frac{c}{d} \\ \text{c) } \frac{a + b + c}{a + b + c} = 1 & \text{d) } \frac{a \cdot b + a \cdot d}{a} = \frac{a(b + d)}{a} = \underline{\underline{b + d}} \\ \text{e) } \frac{a \cdot b \cdot c}{d \cdot e \cdot f} \rightarrow \text{bleibt} & \text{f) } \frac{a \cdot b \cdot c}{c \cdot d \cdot e} = \frac{a \cdot b}{d \cdot e} \end{array}$$

17. Kürze vollständig !

$$\begin{array}{l} \text{a) } \frac{y^4 - y^2}{y^2 + 2y + 1} = \frac{y^2(y^2 - 1)}{(y + 1)^2} = \frac{y^2(y + 1)(y - 1)}{(y + 1)^2} = \underline{\underline{\frac{y^2(y - 1)}{y + 1}}} \\ \text{b) } \frac{-x^2 + 5x - 4}{16 - 8x + x^2} = \frac{-(x^2 - 5x + 4)}{16 - 8x + x^2} = \frac{-(x - 4)(x - 1)}{(x - 4)^2} = \frac{-(x - 1)}{(x - 4)} = \underline{\underline{\frac{-x + 1}{x - 4}}} \\ \text{c) } \frac{-36x^2y}{12x^2y - 60xy} = \frac{-36x^2y}{12xy(x - 5)} = \underline{\underline{\frac{-3x}{x - 5}}} \\ \text{d) } \frac{2y + 2}{5y + 5} = \frac{2(y + 1)}{5(y + 1)} = \underline{\underline{\frac{2}{5}}} \\ \text{e) } \frac{rs - rt}{su - tu} = \frac{r(s - t)}{u(s - t)} = \underline{\underline{\frac{r}{u}}} \\ \text{f) } \frac{p^3 - p^2}{p^3 + p^2} = \frac{p^2(p - 1)}{p^2(p + 1)} = \underline{\underline{\frac{p - 1}{p + 1}}} \\ \text{g) } \frac{2s + 2t}{s^2 - t^2} = \frac{2(s + t)}{(s + t)(s - t)} = \underline{\underline{\frac{2}{s - t}}} \\ \text{h) } \frac{3v + 9}{5v^2 - 45} = \frac{3(v + 3)}{5(v^2 - 9)} = \frac{3(v + 3)}{5(v + 3)(v - 3)} = \underline{\underline{\frac{3}{5(v - 3)}}} \end{array}$$

- i) $\frac{10x + 25y}{4x^2 + 20xy + 25y^2} = \frac{5(2x + 5y)}{(2x + 5y)^2} = \frac{5}{2x + 5y}$
- j) $\frac{x^2 - 2xy + y^2 - z^2}{x^2 - y^2 - 2yz - z^2} = \frac{(x - y)^2 - z^2}{x^2 - (y^2 + 2yz + z^2)} = \frac{(x - y)^2 - z^2}{x^2 - (y + z)^2} = \frac{(x - y + z)(x - y - z)}{(x + y + z)(x - y - z)} = \frac{x - y + z}{x + y + z}$
- k) $\frac{x^2 + 7x + 10}{x^2 - 25} = \frac{(x + 5)(x + 2)}{(x + 5)(x - 5)} = \frac{x + 2}{x - 5}$
- l) gestrichen
- m) $\frac{x^2 - x - 2}{x^2 + 3x + 2} = \frac{(x - 2)(x + 1)}{(x + 2)(x + 1)} = \frac{x - 2}{x + 2}$
- n) gestrichen
- o) gestrichen
- p) $\frac{x - y}{y - x} = \frac{-(-x + y)}{y - x} = \underline{\underline{-1}}$
- q) $\frac{1 - a}{4a - 4} = \frac{1 - a}{4(a - 1)} = \frac{-(-1 + a)}{4(a - 1)} = \underline{\underline{-0.25}}$
- r) $\frac{x^2 + 2x - 63}{7 - x} = \frac{(x + 9)(x - 7)}{7 - x} = \frac{(x + 9)(x - 7)}{-(-7 + x)} = -(x + 9) = \underline{\underline{-x - 9}}$
- s) $\frac{64c^2 - 176c + 121}{121 - 64c^2} = \frac{(8c - 11)^2}{(11 + 8c)(11 - 8c)} = \frac{(8c - 11)^2}{-(11 + 8c)(-11 + 8c)} = \frac{8c - 11}{-(11 + 8c)} = \underline{\underline{\frac{8c - 11}{-8c - 11}}}$
- t) $\frac{r^2 - 2rs + s^2}{5rs - 5r^2} = \frac{(r - s)^2}{5r(s - r)} = \frac{(r - s)^2}{-5r(-s + r)} = \frac{(r - s)}{-5r} = \underline{\underline{\frac{s - r}{5r}}}$